Equivalent Plate Analysis of Aircraft Wing Box Structures with General Planform Geometry

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Described is a new equivalent plate analysis formulation that is capable of modeling aircraft wing structures with a general planform such as cranked wing boxes. Multiple trapezoidal segments are used to represent such planforms. A Ritz solution technique is used in conjunction with global displacement functions that encompass all the segments. This Ritz solution procedure is implemented efficiently into a computer program so that it can be used by rigorous optimization algorithms for application in early preliminary design. A direct method to interface this structural analysis procedure with aerodynamic programs for use in aeroelastic calculations is described. This equivalent plate analysis procedure is used to calculate the static deflections and stresses and vibration frequencies and modes of an example wing configuration. The numerical results are compared with results from a finite element model of the same configuration to illustrate typical levels of accuracy and computation times resulting from use of this procedure.

Nomenclature

$a,b,c,$ e,f,g	= planform dimensions (see Fig. 3)
A	= area of rib or spar cap
C_i	= coefficient of polynomial displacement function
D_{ij}	= orthotropic plate stiffnesses
E	= modulus of elasticity
F_i	= concentrated force at point i
h	= wing box depth
K	= stiffness matrix
ℓ	= coordinate along length of rib or spar cap
L	= length of rib or spar cap
m	= distributed mass
M	= mass matrix
M_i	= concentrated mass at point i
p	= distributed load or pressure
P	= applied load vector
t	= thickness of cover skin layer
V,Q,T	= energy terms [see Eq. (3)]
W	= wing deflection
W_i	= displacement function
x,y	= global chordwise and spanwise coordinates, respectively
X_i, Y_i	= polynomials in x and y for defining displacement functions
ξ,η	= local nondimensional chordwise and spanwise coordinates, respectively

Introduction

SIMPLIFIED beam or plate models of aircraft wing structures are often used for analysis during early preliminary design. For example, such models are usually required for structural optimization when static or dynamic aeroelastic constraints are considered.[1] An equivalent plate model of the wing structure is used in the aeroelastic tailoring and structural optimization (TSO) computer program.[2,3] This program has had widespread use for aeroelastic tailoring of composite wings.[4] However, the structural analysis formulation used in TSO is limited to trapezoidal planforms.

The present paper describes a new equivalent plate analysis formulation that is capable of modeling aircraft wing structures with general planform geometry such as cranked wing boxes. The planform geometry of such wing boxes is defined by multiple trapezoidal segments. The order of the polynomials used to define the wing depth and cover skin layer thicknesses can be specified by the analyst. This new formulation provides a significantly improved structural modeling capability and the analysis procedure has been implemented efficiently so that it can be used by rigorous optimization algorithms for application in early preliminary design.

This paper contains a description of the new analytical formulation along with the methods used for efficient implementation of these analysis procedures into a computer program. This equivalent plate analysis procedure is applied to an example wing configuration to calculate static deflections and stresses and vibration frequencies and modes. The numerical results are compared with results from a finite element model of the same configuration to illustrate typical levels of accuracy and computation times resulting from use of this procedure.

Analytical Modeling

The wing box structure is represented as an equivalent plate in this formulation. Planform geometry of this equivalent plate is defined by multiple trapezoidal segments, as illustrated by the two-segment box in Fig. 1a. A separate local coordinate system is associated with each segment. These local coordinates are nondimensionalized such that ξ refers to a fraction of the local chord and η refers to a fraction of the span for a given segment, as indicated in Fig. 1b. The subscripts on the ξ and η coordinates, shown in Fig. 1 to refer to a particular segment, are omitted in the remainder of this paper since the development of the analysis method is described for a typical segment.

The cross-sectional view of a typical segment shown in Fig. 2 illustrates the analytical modeling of the wing box structure. The depth of the structural box varies over the planform of each segment and is expressed as a polynomial in the nondimensional coordinates ξ and η as

$$h_j(\xi,\eta) = h_{00} + h_{10}\xi + h_{20}\xi^2 + h_{01}\eta + \ldots + h_{mn}\xi^m\eta^n \quad (1)$$

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The coefficients h_{mn} are constants defined by the analyst for each segment. The cover skins consist of orthotropic layers with the thickness of each layer being defined independently by the analyst, again in the polynomial form

$$t_j(\xi,\eta) = t_{00} + t_{10}\xi + t_{20}\xi^2 + t_{01}\eta + \ldots + t_{mn}\xi^m\eta^n \qquad (2)$$

The properties of the layers can be defined to represent wing skins (stiffened panels or composite laminates). Orientation of the stiffness properties, along with the thickness, is specified for each layer and the layer orientations and thicknesses can be different in different planform segments. The upper and lower skins and, hence, the corresponding layers are assumed to be symmetric about the midplane of the wing. The degree of the polynomials in Eqs. (1) and (2) are specified by the analyst.

Rib and spar caps are represented as axial elements continuously attached to the skin. These caps may be positioned arbitrarily within a segment by specifying the locations of their end points. The axial stiffness of each cap can have a linear variation along its length.

For static analysis, loading is applied to the wing box as concentrated forces or distributed loads. Mass properties for dynamic analysis are defined by concentrated or distributed quantities.

The specification of model characteristics as continuous distributions in polynomial form requires only a small fraction of the volume of input data for a corresponding finite element structural model where geometry and stiffness properties are specified at discrete locations. The resulting reduction in model preparation time is important during early preliminary design when many candidate configurations are being assessed. Also, the geometric locations of the rib and spar caps, the mass quantities, and the applied loadings can be independently defined, i.e., they are not referenced to a set of joint locations as in a finite element model. The ease of relocating these quantities without disrupting other aspects of the model is important during early preliminary design when such changes often occur. Finally, the polynomial description of model characteristics lends itself to use with optimization algorithms since the polynomial coefficients can be used directly as design variables.

Ritz Solution Technique

The Ritz method is used to obtain an approximately stationary solution to the variational condition on the energy of the wing box structure and applied loading. This method is a classical approach in structural analysis and details of its application to a single-segment trapezoidal wing planform are described in Ref. 2. Herein, a brief outline of the general technique is given and the methods used to handle planforms with multiple segments are discussed more thoroughly.

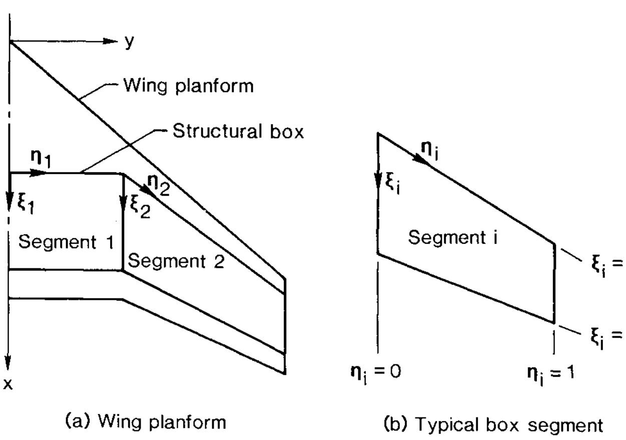

Fig. 1 Coordinate systems used to define wing box structure.

The total energy E associated with the analytical model used is

$$E = V + Q - T \qquad (3)$$

where V is the potential energy of the structure in bending, Q the potential energy of the lateral loads moving through the bending deflections, and T the kinetic energy associated with masses.

These energy terms can be expressed as a function of the bending deflection of the wing structure W, as shown in Table 1. In this application of the Ritz approach, the wing deflection W is assumed to be the sum of contributions C_i from a set of specified displacement functions W_i,

$$W = C_1 W_1 + C_2 W_2 + C_3 W_3 + \ldots + C_n W_n \qquad (4)$$

The Ritz solution procedure is used to determine the numerical values of the set of unknown coefficients C_i that minimizes the total energy E. The total energy E is a function of the wing deflection W and hence can be expressed in terms of the unknown coefficients C_i. The extremum principle stating that E is stationary with respect to C_i, expressed as $dE/dC_i = 0$, produces a system of n simultaneous equations. These equations are expressed in matrix form as

$$[K]\{C_i\} - \omega^2 [M]\{C_i\} - \{P_i\} = 0 \qquad (5)$$

The stiffness and mass matrices $[K]$ and $[M]$ are produced from the energy expressions V and T shown in Table 1. Substitution of the expression for deflection given in Eq. (4) into the expressions for V and T gives a quadratic form of the displacement functions and associated coefficients,

$$\sum_i \sum_j C_i C_j W_i W_j$$

Differentiation of the energies, d^2V/dC_i^2 and d^2T/dC_i^2, produces the stiffness and mass matrices $[K]$ and $[M]$ corresponding to the number of displacement functions used. Each term in these matrices corresponds to a product of displacement functions $W_i W_j$ and associated stiffness or mass quantities. These terms are produced by evaluating the integral expressions shown in Table 1. To complete the matrices, these calculations are repeated for all combinations of the

Table 1 Energy expressions used in Ritz analysis

Quantity	Energy expression
Plate	$V = \frac{1}{2}\int\int_A (D_{11} W_{,xx}^2 + 2D_{12} W_{,xx} W_{,yy} + D_{22} W_{,yy}^2 + 4D_{16} W_{,xy} W_{,xx} + 4D_{26} W_{,xy} W_{,yy} + 4D_{66} W_{,xy}^2)\,dA$
Rib or spar cap	$V = \frac{1}{2}\int_0^L EAh^2 W_{,\ell\ell}\,d\ell$
Distributed load	$Q = -\int\int_A pW\,dA$
Concentrated load	$Q = -\sum_{i=1}^{N} F_i W\vert_{x_i,y_i}$
Distributed mass	$T = \frac{1}{2}\omega^2 \int\int_A mW^2\,dA$
Concentrated mass	$T = \frac{1}{2}\omega^2 \sum_{i=1}^{N} M_i W^2\vert_{x_i,y_i}$

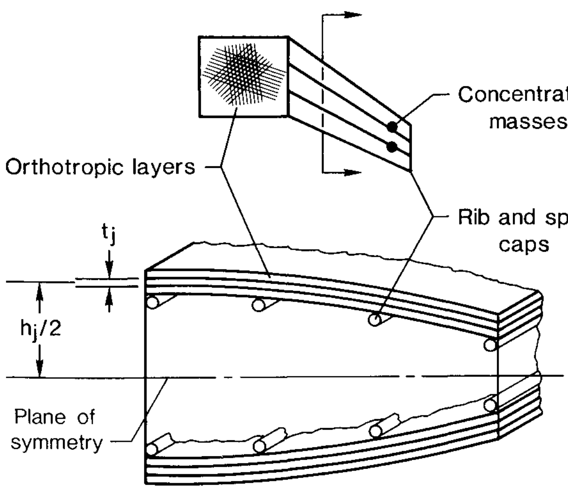

Fig. 2 Analytical modeling of wing box structure.

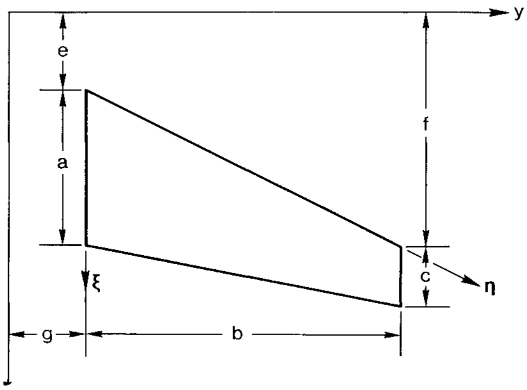

Fig. 3 Planform geometry variables for typical segment.

displacement functions. The energy expression Q is a linear form of C_i and the differentiation $\mathrm{d}Q/\mathrm{d}C_i$ produces a load vector $\{P_i\}$ with each term corresponding to a displacement function.

Energy Expressions

Expressions of the energies that are used in this analysis are given in Table 1. Evaluation of these integral expressions using the assumed displacement functions W_i produce the terms in $[K]$, $[M]$, and $\{P_i\}$. The D_{ij} terms in the expression for the plate are the anisotropic plate bending stiffnesses. These bending stiffness terms D_{ij} are polynomials, which are calculated from the depth and thicknesses given in Eqs. (1) and (2) along with the orthotropic material properties of the composite layers. These properties are defined for each plate segment and the integral expressions are evaluated over the planform area of each segment. The displacement derivative terms $W,_{xx}$, $W,_{yy}$, and $W,_{xy}$ are calculated from the assumed displacement functions used in the Ritz analysis. The choice of the displacement functions is discussed subsequently in this section. The integral expressions for the rib and spar caps are evaluated over the length of the caps. The cap area A can vary linearly along the length.

The energy of the applied loads and masses are functions of the values of these quantities and the displacement functions W_i. The distributed quantities are integrated over the appropriate areas and the concentrated, quantities are summed, with the displacement function being evaluated at the locations of the individual forces or masses.

Displacement Functions

In the present formulation, the assumed displacement functions are specified as products of polynomials in the x direction with polynomials in the y direction of the global (x,y) coordinate system

$$W_i = X_i(x)\,Y_i(y) \tag{6}$$

This approach differs from that in Ref. 2 where the displacement functions are expressed in the local trapezoidal system (ξ,η). The global expression for the displacement functions automatically satisfies the continuity requirements across common boundaries of multiple segments, but does not necessarily satisfy the natural boundary conditions along the tip and leading and trailing edges of the wing box. This approach relies on the minimization of energy to provide an approximation to the boundary conditions at these locations.

An alternative approach to handling multiple segments is to specify sets of displacement functions for each segment and develop a method to insure continuity of the functions and their derivatives across adjacent boundaries. Such an alternative would resemble the global element approach.[5] Such an alternative was not pursued in this study, since it appeared that a simpler approach of using global displacement functions would result in a more efficient program. One of the main purposes of this paper is to determine if the level of accuracy of the results is satisfactory for design purposes when global displacement functions are used over multiple segments.

Another aspect of the formulation involved selecting the type of polynomials to be used to form the displacement functions. At one stage in the development of this method, the implementation allowed the analyst to select or input the set of polynomials to be used. The first set tested were the Legendre polynomials. Using these polynomials, the number of terms in the displacement functions containing the higher-degree polynomials becomes large because of the product of all terms in the x direction with all terms in the y direction. This number of terms is compounded since the structural energy expressions contain the displacement function derivatives to the second power.

Use of sets of terms from a power series, i.e., $(x^0, x^1, x^2, \ldots, x^M)$ for $X_i(x)$ and $(y^0, y^1, y^2, \ldots y^N)$ for $Y_i(y)$, for forming the displacement functions was also evaluated. Several alternative implementation methods were tried in an attempt to achieve a high level of computational efficiency using both Legendre polynomials and power series terms. Comparison of numerical results and computational times from this study led to the selection of terms from a power series for use as the assumed displacement functions. This selection was based on the increased computational efficiency (typically a factor of 10) that can be obtained, compared to Legendre polynomials, by taking advantage of the obvious simplifications in calculations that occur with only one term, $x^m y^n$, in each displacement function. However, this selection results in an upper limit on the degree of the power series terms that can

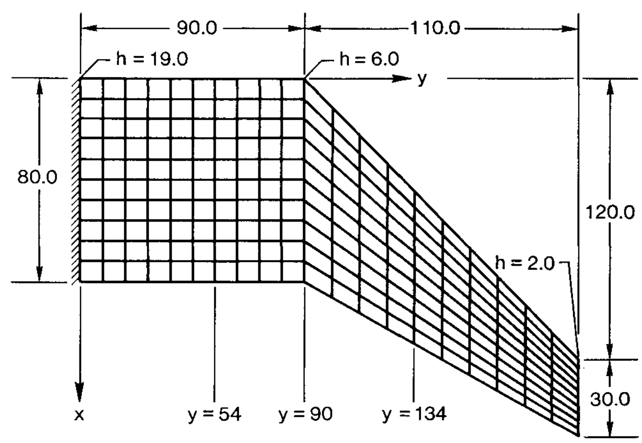

Fig. 4 Planform of example wing box.

be specified because of the ill-conditioning of the resulting set of equations. This ill-conditioning is manifested when the higher-degree terms produce nearly linear dependent equations. The upper limit is reached when the library subroutines used for solution of these equations terminate with a message to indicate excessive numerical error. Typical upper limits were found to be fifth degree in x and eighth degree in y for static solutions and fourth degree in x and seventh degree in y for vibration solutions. The levels of accuracy of results obtained with these degrees of globally defined power series terms are presented in a subsequent section.

Evaluation of Integrals

Plates

The terms in the stiffness matrix $[K]$ of Eq. (5) are produced by evaluating the integral expressions from the structural energy V in Table 1 as described earlier in the general discussion of the Ritz solution technique. The evaluation of the integral expressions for the plate requires that the displacement functions be expressed in terms of the local coordinate system of each plate segment by applying the coordinate transformations

$$x = e + a\xi + (f-e)\eta + (c-a)\xi\eta \tag{7}$$

$$y = g + b\eta \tag{8}$$

The coordinate transformation of the differential area is given by the determinant of the Jacobian as

$$\mathrm{d}x\mathrm{d}y = [ab + (c-a)b\eta]\,\mathrm{d}\xi\mathrm{d}\eta \tag{9}$$

The planform variables for each segment that are used in these transformations are shown in Fig. 3. The anisotropic plate bending stiffnesses D_{ij} are also polynomials in ξ and η. Hence, the terms in the integrand of the expression for plate energy (products of D_{ij} with the transformed displacement derivatives and differential area) are given by power series expressions in ξ and η. These expressions can be integrated over each segment using exact, closed-form expressions to produce the plate segments contributions to the stiffness matrix.

Rib and Spar Caps

The coordinates (x_1,y_1) and (x_2,y_2) and the corresponding cross-sectional areas A_1 and A_2 are specified at the ends of each rib and spar cap. The expression for energy of the caps is given in Table 1. Coordinate transformation equations between the global (x,y) system and the local coordinate ℓ along the length of a cap are

$$x = x_1 + (\ell/L)(x_2 - x_1) \tag{10}$$

$$y = y_1 + (\ell/L)(y_2 - y_1) \tag{11}$$

The curvature along the length of a cap is expressed as

$$W,_{\ell\ell} = W,_{xx}\left(\frac{\mathrm{d}x}{\mathrm{d}\ell}\right)^2 + W,_{yy}\left(\frac{\mathrm{d}y}{\mathrm{d}\ell}\right)^2 + 2W,_{xy}\left(\frac{\mathrm{d}x}{\mathrm{d}\ell}\right)\left(\frac{\mathrm{d}y}{\mathrm{d}\ell}\right) \tag{12}$$

where $\mathrm{d}x/\mathrm{d}\ell = (x_2 - x_1)/L$ and $\mathrm{d}y/\mathrm{d}\ell = (y_2 - y_1)/L$. The cap area is taken to vary linearly along the length, $A = A_1 + (\ell/L)(A_2 - A_1)$. The depth of the wing h is expressed in terms of ξ and η in Eq. (1). Expressing the depth h in the local coordinate system of the cap results in a complicated (not simple power series) integral equation. Therefore, evaluation of the integral expressions for the caps are approximated using the trapezoidal rule with the number of intervals used for numerical integration along the cap length specified by the analyst.

Mass Properties

The mass properties associated with the analytical model are defined as being distributed over the wing planform and/or concentrated at specified points. Distributed masses are often defined directly as a function of ξ and η, e.g., the mass per unit area of the cover skin is given by the product of the material density and the skin thickness given in Eq. (2). Evaluation of the integral expressions for such distributed masses is performed using the same exact, closed-form expressions that are used for the plate stiffness integrals. Contributions of the concentrated masses to the mass matrix are the products of each mass with the quadratic form of the displacement functions evaluated at the location of that particular mass.

Applied Loads

The expressions of energy for distributed and concentrated applied loads are similar to the expressions for masses, except that the loading expressions are linear functions of the wing deflection W. Application of the Ritz method to this linear form results in a load vector for each set of applied loads. For aircraft wings, the distribution of aerodynamic loading is usually calculated as a table of pressure coefficients at a specified set of chord and semispan stations. These pressures can be converted to a set of concentrated loads by multiplying each pressure coefficient by its associated area. These concentrated forces are then multiplied by the values of each displacement function at the point of load application to give the appropriate terms in the load vector. The continuous definition of the displacement functions expedites this process, since the values of displacements can be calculated directly at the desired points of the aerodynamic grid. Hence, the transformation process that must be performed between the aerodynamic grid and structural joints when finite element structural modeling is used is not required. This continuous definition of displacement functions provides a direct method to interface this equivalent plate structural analysis procedure with aerodynamic programs for use in aeroelastic calculations.

Implementation of Method

Implementation of this Ritz solution method into a computationally efficient computer program is an important facet of this development. Clearly, the terms associated with calculating coefficients of a stiffness matrix are algebraically cumbersome and tedious to manipulate. This is especially true for an anisotropic plate segment. Therefore, several alternative strategies for organizing and performing the calculations were explored before reaching the following methods of implementation.

All quantities in this equivalent plate formulation are represented as polynomials containing the sum of a sequence of terms composed of a coefficient and two variables with integer exponents. These polynomials are represented as matrices of the coefficients; each coefficient is located with the row index being one greater than the exponent of the first variable and the column index being one greater than the exponent of the second variable. The shifting by one is necessary to handle the constant terms (variables to the zero power). These matrices are stored as vectors with the first two entries containing the total number of rows and number of columns in the matrix. This representation allows the computations to be independent of the order and type (e.g., power series, Legendre, etc.) of polynomials used to represent wing box depth, thicknesses of the cover skin layers, and assumed displacement functions.

A special library of subroutines was developed to perform all of the mathematical operations on these polynomials. These operations include addition, subtraction, multiplication, differentiation, integration, and evaluation at a point of polynomials representing quantities in two dimensions. This library of subroutines is used to generate the terms in matrix equation [Eq. (5)] by forming and evaluating the integral and summation expressions of Table 1 in the manner described in the previous section.

The matrix representation of the polynomials and the special mathematical library of subroutines for operating on the polynomials are key tools used for efficient implementation of the equivalent plate analysis procedure. However, a detailed description of how these tools were actually represented and applied to form a computer program for this particular application is beyond the scope of this paper.

Typical Application and Results

Analytical Modeling

The planform of the wing box used to evaluate this new formulation is shown in Fig. 4. This example is representative of a typical fighter aircraft wing box and provides a model with two plate segments. The configuration provides a good test case, since twisting behavior is dominant in the inner segment and bending behavior dominant in the outer segment. The numerical results from this model indicate how well the single, global set of displacement functions represents the structural response of this cranked wing box.

The wing box depth has a different linear spanwise variation in each segment and the depth is constant in the chordwise direction. The cover skin is a single layer of constant thickness aluminum. Clamped boundary conditions are applied to the wing box at the aircraft centerline.

Results from the equivalent plate analysis are compared with corresponding results from the EAL finite element analysis program.[6] The EAL model is built up of membrane rib, spar, and cover elements with the grid of cover elements shown in Fig. 4 giving 1320 degrees of freedom in the finite element analysis. Displacement functions used in the equivalent plate analysis contained exponents 0–4 in the chordwise x direction (five terms) and exponents 2–7 in the spanwise y direction (six terms) resulting in 30 unknown coefficients that correspond to the generalized degrees of freedom.

Numerical Results

Static Analysis

For numerical testing of this method, a uniform pressure loading is applied to the wing box. The resulting static displacements along the leading and trailing edges of the wing box are shown in Fig. 5; the solid lines indicate results from the equivalent plate analysis and the individual symbols indicate data from the finite element analysis. Displacements from the two analysis methods agree within 1% throughout the wing box.

The distribution of stresses across the wing chord are shown in Figs. 6–8 for three different semispan locations ($y = 54$, 90, and 134 as indicated in Fig. 4). In general, the agreement in stresses is good, except in the region of the trailing edge at the inboard of the wing box crank. As would be expected, the equivalent plate technique provides a good representation of stresses in the outboard portion of the wing box where the stress gradients are small, but is less accurate in the inboard region where larger gradients occur from the crank in the trailing edge and clamped boundary conditions at the wing root.

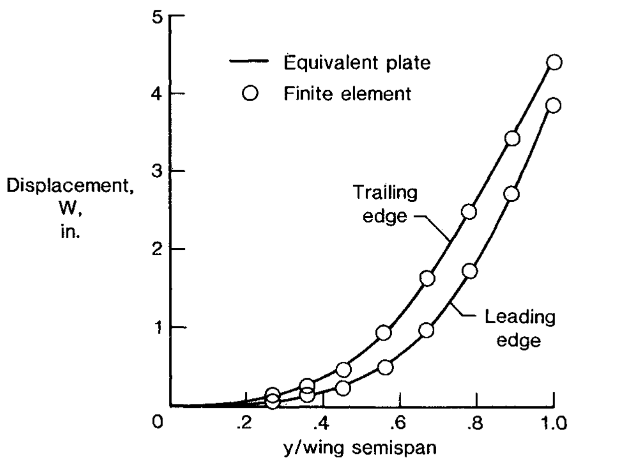

Fig. 5 Displacements along leading and trailing edges of wing box.

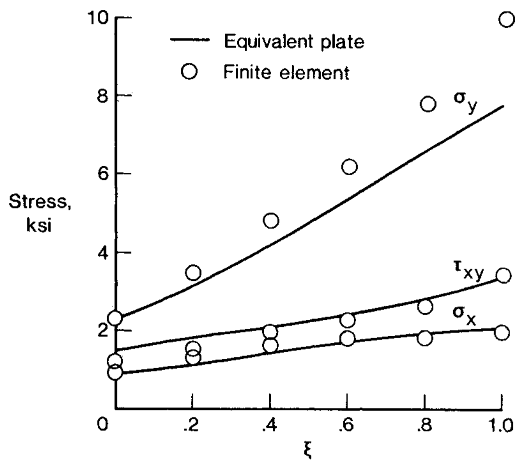

Fig. 7 Stress distributions at $y = 90$ (crank location).

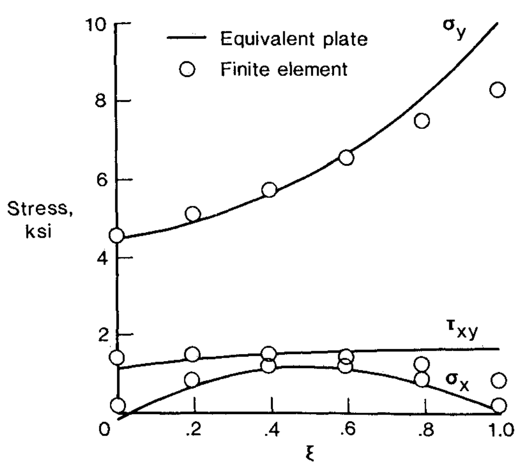

Fig. 6 Stress distributions at $y = 54$.

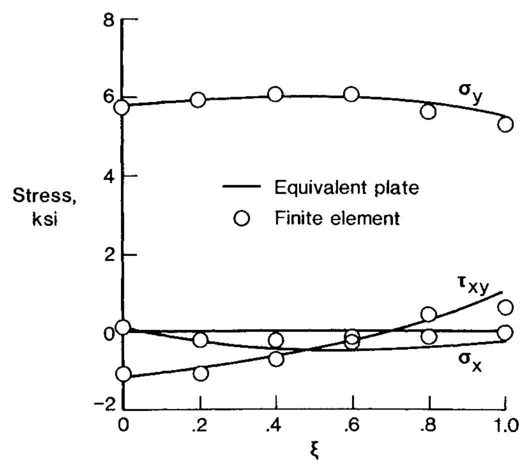

Fig. 8 Stress distributions at $y = 134$.

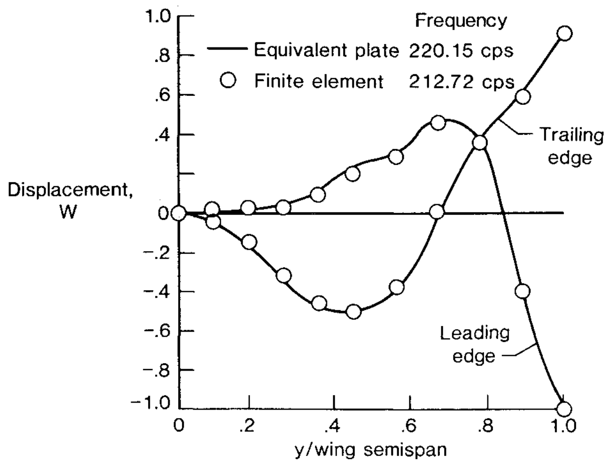

Fig. 9 Sixth vibration mode shape.

Table 2 Comparison of natural frequencies from vibration analysis

No.	Finite element, cycle/s	Equivalent plate, cycle/s	Difference, %
1	14.58	14.76	1.2
2	48.52	49.10	1.2
3	97.22	99.99	2.9
4	113.99	117.53	3.1
5	174.73	181.22	3.7
6	212.72	220.14	3.5
7	277.38	294.80	6.3

Table 3 Comparison of computer times

	Equivalent plate		Finite element	
Task	Increment	Total	Increment	Total
Form stiffness matrix	2.28	2.28	86.07	86.07
Static solution	1.21	3.49	30.58	116.65
Displacements and stresses	1.25	4.74	9.84	126.49
Vibration analysis	2.73	7.47	352.33	478.82

Vibration Analysis

Natural vibration frequencies and mode shapes for the cranked wing box are calculated using both the equivalent plate analysis and the finite element analysis. A comparison of the first seven natural frequencies is given in Table 2. The percent difference is small (1.2%) for the first frequency and this difference increases with increase in frequency. The displacements along the leading and trailing edges of the wing box for the sixth vibration mode are presented in Fig. 9. This mode shape is dominated by torsion of the wing and the agreement between results from the equivalent plate analysis and finite element analysis is excellent. The results for the other calculated mode shapes exhibit a comparable level of agreement.

Computation Times

A comparison of the computational times required for the equivalent plate analysis and the finite element analysis is given in Table 3. The number of CPU seconds is given for selected major tasks involved in a static and vibration analysis. An accumulation of these incremental times is also given. All calculations were performed on a CDC Cyber-173 computer. Comparison of the total time required to produce displacements and stresses from a static analysis indicate that the equivalent plate analysis is a factor of 30 faster than the finite element analysis. For vibration analysis, the corresponding comparison is a factor of 60. The times for the equivalent plate analysis did not include the 7.89 s of CPU time required to generate the integral tables. These tables are independent of the stiffness orientation and thickness of the layers in the cover skins and do not change during an optimization process. The tables can be generated in a separate computer run and saved for subsequent use.

The computer times are for a wing deflection expression with 30 unknown coefficients (degrees of freedom) corresponding to assumed displacement functions that contained terms up to fourth power in x and seventh power in y. These times are reduced when fewer displacement functions are used, but some loss in accuracy is incurred. Although it is problem dependent, the upper limit on displacement functions was found to be about seventh degree without encountering ill-conditioning problems on CDC (60 bit words) computers. Since the computer times for 30 degrees of freedom are relatively small, use of approximately this number of displacement functions is recommended.

In addition to providing desirable computational speed, the equivalent plate analysis computer program has moderate memory requirements. Implementation methods that keep the memory requirements small are important for effective interactive operation of the resulting computer program or for effective coupling of the analysis procedure with an optimization procedure.

Conclusions

A description is given of a new equivalent plate analysis formulation that is capable of modeling aircraft wing structures with general planform geometry such as cranked wing boxes. Methods are discussed for implementing these general procedures into a computer program that is simply organized and computationally efficient and, hence, desirable for use in early preliminary design.

Some typical numerical results are presented from application of the procedure to a cranked wing box. Comparison of these results with corresponding results from a finite element analysis program indicate that good agreement, generally less than 5% difference, is obtained for both static displacements and vibration frequencies and mode shapes. In general, the agreement in stresses is good except in the region of the trailing edge at or inboard of the wing box crank. The computation time required by the equivalent plate analysis to generate these results is a factor of 30 less than a corresponding finite element analysis for a static analysis and a factor of 60 less for a vibration analysis.

In summary, application of the new equivalent plate analysis formulation to a cranked wing box is shown to produce results with levels of accuracy approaching that of a finite element analysis in significantly less computation time. Hence, this formulation provides a structural analysis capability with desirable characteristics for combination with aerodynamic analyses and rigorous optimization procedures to perform aeroelastic tailoring of the cranked wing box structures.

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